A DYNAMIC THERMAL IDENTIFICATION METHOD: PART I – NON CONDUCTOR SOLID MATERIALS

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ABSTRACT

This two-part study presents experimental methodologies for the determination of thermal properties. Part I describes a thermal identification technique to be applied in non conductor solid materials, while Part II deals with the thermal identification of conductor solid materials. In Part I the technique, that estimates the thermal diffusivity, α, and the thermal conductivity, λ, simultaneously is based on signal treatment in both frequency and time domains of a same data. In the frequency domain, a dynamic model is defined by combining two heat flux and two superficial temperature. The thermal diffusivity is, then, obtained by identifying the phase of the frequency response of the model performed numerically. Once the thermal diffusivity is known the same data can be used to obtain the thermal conductivity in the time domain. In this case a least square function of measured and calculated temperature is used. The independent process for the thermal properties identification assures the obtained values of α and λ to be unique. A comparison with the Flash and the Guarded Hot Plate methods for a PVC (Polyvinyl chloride) sample gives a deviation of 3.22 % and 3.30 % for α and λ, respectively.

NOMECLATURE

F	frequency, Hz
FFT	Fast Fourier Transform
FM	Flash Method
GF	Green's Function
GHPM	Guarded Hot Plate Method
$H(f)$	frequency response function, m^2K/W
$Im(S_{xy})$	imaginary component of the cross-spectral density function, K^2
n	total number of time measurements
$Re(S_{xy})$	real component of the cross-spectral density function, K^2
s	the number of sensors
S_{mq}	least square objective function, K^2
S_p	phase objective function, rad^2
S_{xx}	autoespectral density function of X(t), K^2
S_{yy}	autoespectral density function of Y(t), K^2
S_{xy}	cross-spectral density function, K^2
$\lvert S_{xy} \rvert$	is the modulus of S_{xy}
t	time, s
T_o	initial temperature, °C
T_i	temperature, °C
x,*y*	cartesian coordinates, m
$X(t)$	input signal in time domain, W/m^2
$Y(t)$	output signal in time domain, °C
$\mathbf{X}(f)$	input signal in frequency domain
$\mathbf{Y}(f)$	output signal in frequency domain
$Z(f)$	impedance function
$\lvert Z(f) \rvert$	modulus of $Z(f)$

GREEK SYMBOLS

α	thermal diffusivity, m^2/s
ϕ_i	heat flux, W/m^2
φ	phase factor
φ_{xy}	is the phase factor of S_{xy}
λ	thermal conductivity, W/mK
θ	temperature difference, °C

Subscripts

t	relative to calculated data
e	relative to experimental data
m	relative to integer variables
1	relative to the thermocouple 1
2	relative to the thermocouple 2

INTRODUCTION

A considerable effort has been devoted for the fulfillment of the ever-growing demand for new materials with relevant application in engineering, especially in the evaluation of insulation material performance. In this case, the characterization of thermophysical properties as thermal diffusivity, α, and thermal conductivity, λ, is essential for the correct prediction of the thermal behavior of these materials. Some experimental methods have been used for determining these properties as the hot wire and flash methods. Blackwell [1] presents the hot wire technique for the measurement of the thermal conductivity. The technique requires inserting a probe inside the sample, and this appear to be the main difficulty of the method when applied to solid materials. Variations of this method have been used in recent works. For example, in Miyamura and Susa [2] for λ determination of liquid gallium and in Nahor *et al.* [3] to determine the optimal sensor in hot wire probe set up in foods. Parker *et al.* [4] have developed one of the more employed methods for measuring α of solid materials. This method involves exposing a thin slab of the material to a very short pulse of radiant (or other form) energy. The thermal diffusivity is determined through the identification of the time when the rear surface of the sample reach half of the maximum temperature rise. The use of flash method to measure α has been employed in countless papers as in Mardolcar [5] in rocks at high temperature and Eriksson *et al.* [6] in Liquid silicate melts. It should be observed that in both hot wire and flash methods only one property can be obtained with precision. The parameter estimation technique Beck & Arnold [7] can also be used to estimate simultaneously the thermal properties α and λ for different types of material. Although some efforts should be done to avoid low sensitivity regions for obtaining both properties with confidence. Guimarães *et al.* [8] and Lima e Silva *et al.* [9] have proposed the simultaneous estimation in independent ways. They use two independent objective functions for each one of the properties, thermal conductivity and thermal diffusivity, what assures the estimated values to be unique. In the first technique the properties are estimated in the frequency domain while the time domain case is used in the last technique. However, both techniques have some difficulties. In Ref. [8] the analytical dependence of $Z(f)$ becomes difficult to apply two or three-dimensional models while the problem of low sensitivity in α estimation must be minimized in Ref. [9]. One way to alleviate these difficulties is the development of a technique that uses both the time and frequency domains to estimate α and λ.

In contrast of previous work of Guimarães *et al.* [8] that is based on one-dimensional analytical solution, this work not only use the one-dimensional numerical, but also a three-dimensional conception (showed in Part II). The great advantage of this procedure is the easiness in data experimental manipulation. Results of α and λ are in good agreement for two different polymers: Polythene and Polyvinyl chloride.

THEORETICAL FUNDAMENTAL

The technique proposed here is based on the use of an input/output dynamical system (Fig. 1) that is obtained from a thermal model. The thermal model can be given by a one-dimensional model as shown in Fig. (2).

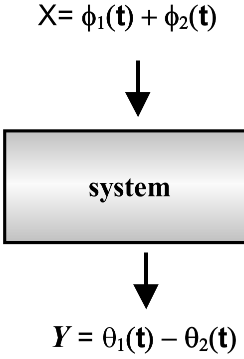


Figure 1. Input/output dynamic system

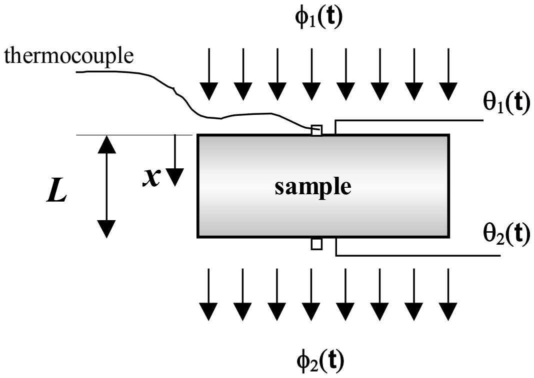


Figure 2.Thermal model scheme

The output signal Y must be obtained by solving the heat diffusion equation with the appropriate boundary conditions. The one-dimensional heat diffusion solution can be analytically obtained just involving the variables ϕ_i and T_i in lower and upper surfaces [8].

The temperature distribution in a finite one-dimensional, constant property body which has an initial temperature distribution T_0, can be described by

$$\frac{\partial^2 T(x,t)}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T(x,t)}{\partial t} \tag{1}$$

subject to the boundary conditions

$$-\lambda\frac{\partial T(x,t)}{\partial x}\bigg|_{x=0} = \phi_1(t) \qquad \lambda\frac{\partial T(x,t)}{\partial x}\bigg|_{x=L} = \phi_2(t) \tag{2}$$

and initial condition

$$T(x,0) = T_0 \tag{3}$$

The solution to the problem given by Eqs. (1)-(3) can be obtained through the use of GF, [10]. In this case the temperature solution is given by

$$T(x,t) - T_0 = \frac{\alpha}{\lambda}\int_{\tau=0}^{t}\left[\int_{S'} G(x/x',t-\tau)\big|_{z'=0}\, dS'\right]\phi(\tau)\ d\tau \tag{4}$$

where $G(x\ /\ x', t\text{-}\tau)$ is the GF associated with the thermal problem. Equation (4) can be rewritten as

$$\theta(x,t) = \int_{\tau=0}^{t} G^{+}(x, t-\tau)\ .\ \phi(\tau)\ d\tau \tag{5}$$

where

$$G^{+}(x,t-\tau) = \frac{\alpha}{\lambda}\int_{S'} G(x/x',t-\tau)\big|_{z'=0}\, dS' \tag{6}$$

Hence for two different positions x_1 and x_2, Eq. (5) is given by

$$\theta(x_1,t) = \theta_1(t) = -\int_{\tau=0}^{t} G_1^{+}(\ t-\tau)\ .\ \phi_1(\tau)\ d\tau \tag{7}$$

and

$$\theta(x_2,t) = \theta_2(t) = -\int_{\tau=0}^{t} G_2^{+}(t-\tau).\phi_2(\tau)\ d\tau \tag{8}$$

If $G_i^{+}(t)$ and $\phi_i(t)$ are functions of t defined for $t > 0$, then Eqs. (7) and (8) represent the convolution integral of these functions named by $G_i^{+} * \phi_i$, hence

$$\theta_1(t) = G_1^{+} * \phi_1(\tau) = \int_{\tau=0}^{t} G_1^{+}(t-\tau).\phi_1(\tau)\ d\tau \tag{9}$$

and

$$\theta_2(t) = G_2^{+} * \phi_2(\tau) = -\int_{\tau=0}^{t} G_2^{+}(t-\tau).\phi_2(\tau)d\tau \tag{10}$$

If the Fourier transform ($\Im$) of convolution of Eqs. (9) and (10) exist, one of its properties can be applied [11] as

$$\Im\left(G_1^{+}(t) * \phi_1(t)\right) = \overline{G}_1^{+}(\ f).\overline{\phi}_1(f) \tag{11}$$

and

$$\Im\left(G_2^{+}(t) * \phi_1(t)\right) = -\overline{G}_2^{+}(\ f).\overline{\phi}_2(f) \tag{12}$$

In this case Eqs. (11) and (12) can be identified in the frequency domain as

$$\overline{\theta}_1(f) = \overline{G}_1^{+}(f).\overline{\phi}_1(f) \tag{13}$$

and

$$\overline{\theta}_2(f) = -\overline{G}_2^{+}(f).\overline{\phi}_2(f) \tag{14}$$

Once the data $\phi_i(t)$ and $\theta_i(t)$ are experimental values, it means discrete values, the Fourier transforms can be performed numerically by using the Cooley-Tukey algorithms FFT [11]. Subtracting Eq. (13) from Eq. (14) gives

$$\overline{\theta}_1(f) - \overline{\theta}_2(f) = Z(f)\ .\left[\overline{\phi}_1(f) + \overline{\phi}_2(f)\right] \tag{15}$$

where *Z*(*f*) is the impedance function, that is equivalent to the frequency response function *H*(*f*) of a Dynamical system given by

$$Z(f) = \bar{G}_1^+(f) - \bar{G}_2^+(f) = \frac{\boldsymbol{Y}(f)}{\boldsymbol{X}(f)} \tag{16}$$

as defined in Guimarães *et al.* [8], it means, considering the Eq. (15) and Fig. (1b) the impedance function can also be written as

$$Z(f) = \frac{\bar{\theta}_1 - \bar{\theta}_2}{\bar{\phi}_1 + \bar{\phi}_2} = \frac{\boldsymbol{Y}(f)}{\boldsymbol{X}(f)} \tag{17}$$

The ***X*(*f*)** and ***Y*(*f*)** data are obtained experimentally by applying the Fourier transform to *X*(*t*) and Y(*t*) signals, it means,

$$\boldsymbol{Y}(f) = \int_0^\infty Y(t) e^{-j 2\pi f t} dt \tag{18}$$

$$\boldsymbol{X}(f) = \int_0^\infty X(t) e^{-j 2\pi f t} dt \tag{19}$$

where $j = \sqrt{-1}$. A more stable impedance function can be obtained by multiplying Eq. (17) by the complex conjugate of ***X***(*f*),

$$Z(f) = \frac{\boldsymbol{Y}(f)\boldsymbol{X}^*(f)}{\boldsymbol{X}(f)\boldsymbol{X}^*(f)} = \frac{S_{xy}(f)}{S_{xx}(f)} \tag{20}$$

where S_{xy} is the cross-spectral density of *X*(*t*) and *Y*(*t*) and S_{xx} is the autoespectral density of *X*(*t*) [11]. In polar form, *Z*(*f*) can be written as

$$Z(f) = |Z(f)| e^{-j\varphi(f)} \tag{21}$$

where $|Z(f)| = |S_{xy}(f)| / S_{xx}(f)$, and $\varphi(f) = \varphi_{xy}(f)$. The auto-spectral density function S_{xx} is a real function. The transformed frequency response in the *f*-*x* plane, is, thus, a complex variable, with a modulus $|Z|$ and a phase factor φ.

THERMAL DIFFUSIVITY ESTIMATION: FREQUENCY DOMAIN

The great convenience of working in the frequency domain is the fact that the phase factor is a function that depends only on the thermal diffusivity α. The basic idea here is the observation that the delay between the experimental and theoretical temperature is an exclusive function of α. So, the minimization of an objective function, S_p, based on the difference between of the experimental and calculated values of the phase is the way to determine the thermal diffusivity. This function can, then, be written by

$$S_p = \sum_{i=1}^{Nf} \left(\varphi_e(i) - \varphi_t(i)\right)^2 \tag{22}$$

where φ_e and φ_t are the experimental and calculated values of the phase angle of *Z* respectively. The theoretical values of the phase angle are obtained from the identification of *Z*(*f*) by Eq. (20). In this case the output ***Y***(*f*) is the Fourier transform of the difference $\theta_1(t)$ - $\theta_2(t)$ obtained by the numerical solution of Eqs. (1)-(3) by using the finite volume method [10]. In fact, this procedure avoids the necessity of an explicit and analytical model of *Z*(*f*). This kind of solution is the base of a 3D model to be applied in metal thermal identification. The input $\phi_1(t)$ + $\phi_2(t)$ responsible for thermal identification can be obtained from either experimental or simulated data. The values of α will be supposed to be those which minimize Eq. (22). In this work this minimization is done by using the golden section method with polynomial approximation [12].

THERMAL CONDUCTIVITY ESTIMATION: TIME DOMAIN

Once the thermal diffusivity value is obtained, an usual objective function based on temperature error can be used to estimate the thermal conductivity. In this case, there are no identifiability problems as just one variable is being estimated. Therefore, the variable λ will be the parameter that minimizes the least square function, S_{mq}, based on the difference between the calculated and experimental temperature defined by

$$S_{mq} = \sum_{j=1}^{s} \sum_{i=1}^{n} \left[\theta_e(i,j) - \theta_t(i,j)\right]^2 \tag{23}$$

The optimization technique used to obtain λ is also the golden section method with polynomial approximation [12].

EXPERIMENTAL APPARATUS

Two polymers samples were used, Polythene and Polyvinyl chloride (PVC), both with thickness of 50 mm and lateral dimensions of 305 x 305 mm. Figure 2 shows the experimental apparatus where at time $t = 0$, the sample is in thermal equilibrium at T_0. At this time, the sample is submitted to a unidirectional and uniform heat flux on its upper surface. The heat is supplied by a 22 Ω electrical resistance heater, covered with silicone rubber, with lateral dimensions of 305 x 305 mm and thickness 1.4 mm. The heat flux are acquired by a transducer with lateral dimensions of 50 x 50 mm, thickness 0.1 mm, and constant time less than 10 ms. The transducer is based on the thermopile conception of multiple thermoelectric junction (made by electrolytic deposition) on a thin conductor sheet Güths [13]. The temperatures are measured using surface thermocouples (type K). The signals of heat flux and temperatures are acquired by a data acquisition system HP Series 75000 with voltmeter E1326B controlled by a personal computer.

Figure 3 – Experimental apparatus used for determining α and λ

RESULTS AND DISCUSSION

Figures (4) and (5) show respectively the evolution of the input signal and the output signal in function of time for one of the experimental of PVC sample.

Once the signals data have the same behavior for both PVC and Polythene, just the PVC test will be presented. Fifty independent runs for PVC and twenty independent runs for Polythene were realized. For both samples 1024 points were taken, where the time intervals, Δt, were 7.034 s for PVC and 6.243 s for Polythene. The time duration of heating, t_h, was approximately 150 s for PVC and 90 s for Polythene with a heat pulse generated by a 40 V (DC) for both samples. As shown in Guimarães *et al.* [8] and Lima e Silva *et al.* [14] the phase factor depends only on α. In this case, analyses in the frequency regions in order to determine the best band for obtaining α are shown. The real and imaginary components of the cross-spectral density are shown in Figs. (6) and (7). The spectral density functions were obtained by averaging.

The cross-spectral density is negligible for frequencies greater than 2.0 x 10^{-03} Hz. In this manner, the frequency range of interest for the generalized impedance is less than 2.0 x 10^{-3} Hz. This is an important result because it establishes the sampling time interval and the total number of experimental points of interest.

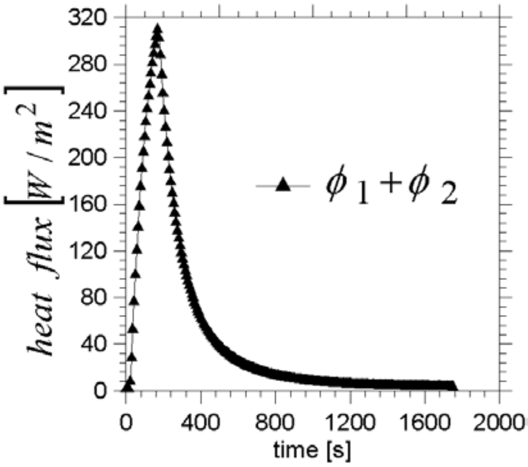


Figure 4 – Evolution of the input signal

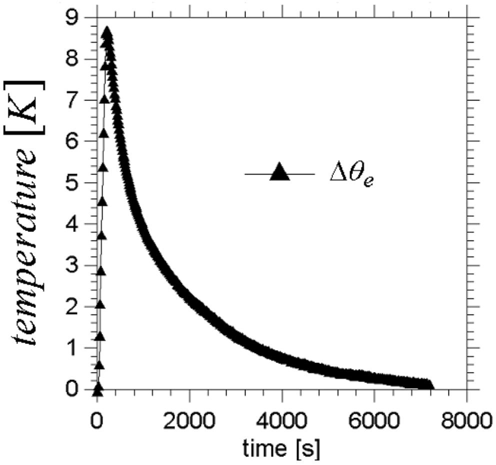


Figure 5 – Evolution of the output signal
$Y(t) = T_1(t) - T_2(t)$

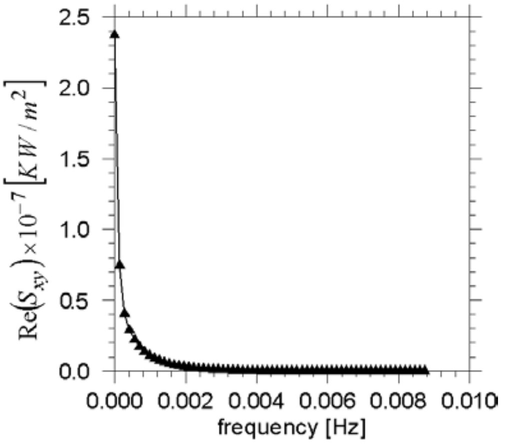


Figure 6 – Real cross-spectral density function

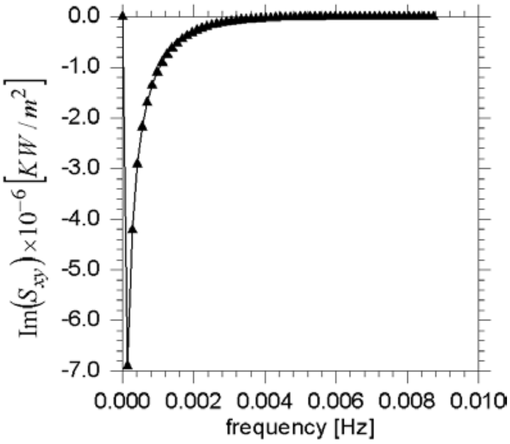


Figure 7 – Imaginary cross-spectral density function, $S_{xy}(f)$

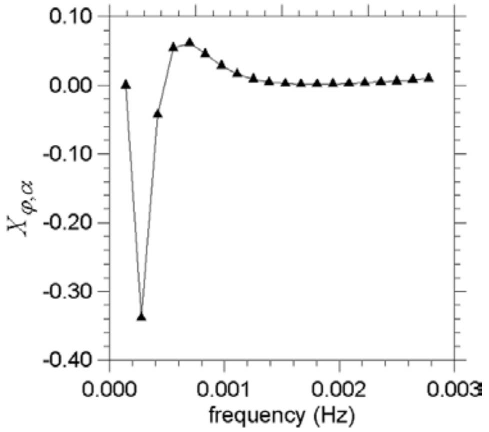


Figure 8 – Scaled sensitivity coefficient related to α

Another important way of analysis is the behavior of the sensitivity coefficients involved in the process. The first sensitivity coefficient analyzed is the $X_{\varphi,\alpha}$ that is defined as the first derivative of phase angle with respect to the parameter α. Figure (8) shows the behavior of $X_{\varphi,\alpha}$ in frequency domain. It can be seen that for frequencies greater than 1.0 x 10^{-3} Hz, $X_{\varphi,\alpha}$ becomes constant and a little contribution is given for the estimation procedure. This fact reduces the analysis band and establishes the interest frequency in values less than 0.001 Hz. The other important coefficient is related with λ in the time domain. Figure (9) presents the $X_{\theta,\lambda}$ coefficient that is defined as the first derivative of the temperature model θ with respect to λ. The high values of this coefficient shows the great advantage of thermal conductivity estimation in time domain.

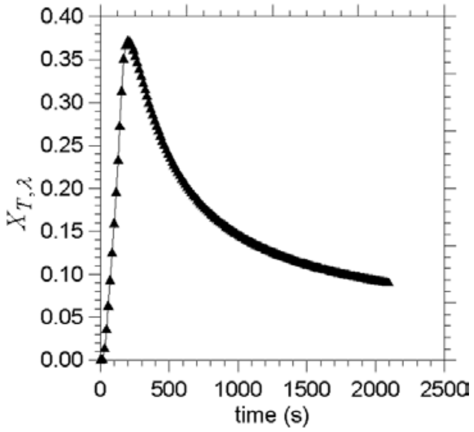


Figure 9 – Scaled sensitivity coefficient related to λ

Table 1. Statistics data to the averaged value of α, (initial value of $\alpha = 1.0 \times 10^{-8}$ m^2/s).

α (m^2/s) x 10^7	Initial S_p x 10^{02}	Final S_p x 10^{07}	σ (m^2/s) x 10^{10}
1.24	6.0	3.6	7.06

Tables (1) and (2) present respectively the value estimated of α and λ for the fifty runs of PVC, with 99.87 % confidence interval. In Table (3) a summary of the simultaneous estimation of α and λ of the PVC sample is presented. In this table, the comparison with the values obtained for α by using the Flash method [15] and λ by using the guarded hot plate method [15] presented errors of 3.22 % and 3.30 % for α and λ, respectively.

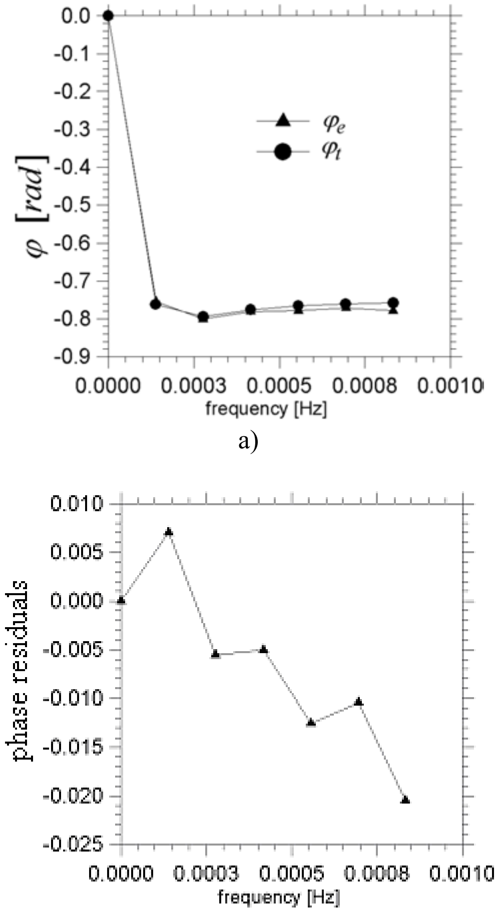
Table 2. Statistics data to the averaged value of λ, (initial value of λ = 0,01 W/m.K).

α (m^2/s) x 10^{07}	λ (W/mK)	Initial S_{mq} 10^{-06}	Final S_{mq}	σ x 10^{05} (W/m.K)
1.24±1.88%	0.152	1.351	5.91	4.9

Table 3. Summary of α and λ for PVC sample

α (m^2/s)x10^7	α (m^2/s) x 10^7 (FM)	λ (W/m.K)	λ (W/m.K) (HPM)
1.24±1.88 %	1.28±3.1 %	0.152±1.1 %	0.157

In Figure (10a) a comparison between experimental and estimated phase factor is presented. It can be observed a very good agreement between them. The Figure (10b) shows the residuals.



a)

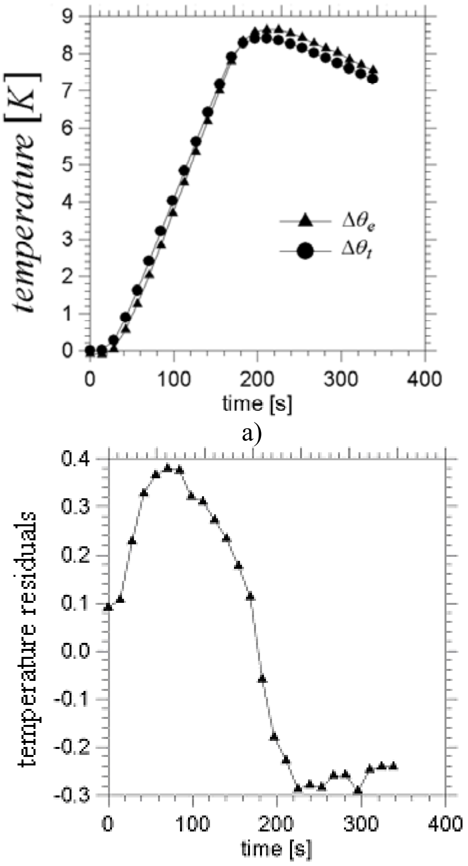
b)

Figure 10. Phase factor: a) experimental and calculated data b) residuals

The comparison between the experimental and estimated temperatures for α = 1.24 x 10^{-07} m^2/s and λ = 0.152 W/m.K is shown in Fig. (11a). Again a good agreement between the data can be observed. It can be noted that the residuals presented in Fig. (11b), are situated in the range of uncertainty measurement of thermocouples, that in this work is ± 0.3 K. Table 4 presents a summary of the simultaneous estimation of α and λ for the Polythene sample with a confidence interval of 99.87 %. For this sample only the reference value for λ obtained by NPL [16] is presented.

Table 4. Summary of α and λ for Polythene sample estimation

α (m^2/s) x 10^7	λ (W/m.K)	λ (W/m.K) (HPM)	Error (%)
2.14±1.15%	0.383±1.68 %	0.389	1.57



a)

b)

Figure 11.Temperature evolution: a)experimental and calculated data b) residual

CONCLUSION

It is proposed in this work a technique to simultaneously estimate the thermal diffusivity and thermal conductivity for two polymers. The results have shown good agreement when compared with literature values. The fact of calculating the phase factor numerically is one of the most important advantages of the technique proposed here. This procedure allows the use of a 3D transient model and consequently its application to conductor material identification. The metal application is shown in Part II of this study.

ACKNOWLEDGEMENT

The authors would like to thank the support of the fomentation organs CAPES, Fapemig and CNPq.

REFERENCES

1. J. H. Blackwell, A transient-flow method for determination of thermal constants for insulating materials in bulk, *Journal of Applied Physics*, **25**, 137 (1954).

2. A Miyamura, and M. Susa, Relative measurements of thermal conductivity of liquid gallium by transient hot wire method, *The Sixteenth European Conference for Thermophysical Properties* (*ECTP2002*), London, UK, (2002).

3. H. B. Nahor, N. Scheerlinck, J. F. Van Impe, and B. M. Nicolaï,, Optimization of the temperature sensor position in a hot wire probe set up for estimation of the thermal properties of foods using optimal experimental design", *Journal of Food Engineering*, **57**, 103 (2003).

4. W. J. Parker, R. J. Jenkins, C.. P. Butler, G. L. and G. L. Abbot, Flash method of determining thermal diffusivity, heat capacity and thermal conductivity", *Journal of Applied Physics*, **32**, 1679 (1961).

5. U. V Mardolcar, Thermal diffusivity of rocks at high temperature by the laser flash technique, *The Sixteenth European Conference for Thermophysical Properties* (*ECTP2002*), London, UK, (2002).

6. R. Eriksson, M. Hayashi, and S. Seetharaman, Thermal diffusivity measurements of liquid silicate melts, *The Sixteenth European Conference for Thermophysical Properties* (*ECTP2002*), London, UK, (2002).

7. J. V. Beck, and K. J Arnold, .*Parameter Estimation in Engineering and Science*, John Wiley and Sons Inc., New York., 1977, p. 503.

8. G. Guimarães, P. C. Philippi, and P. Thery, Use of parameters estimation method in the frequency domain for the simultaneous estimation of thermal diffusivity and conductivity, *Review of Scientific Instruments*, **66**, 2582 (1995).

9. , S. M. M. Lima e M. A. V. Silva Duarte, and G. Guimarães, A correlation function for thermal properties estimation applied to a large thickness sample with a single surface sensor, *Review Scientific Instrument*, **69**, 3290 (1998).

10. M. N. Özisik, *Heat Conduction*, John Wiley & Sons, 2nd ed., New York, 1993, p. 692.

11. J. S. Bendat, and A. G. Piersol, *Analysis and Measurement Procedures*, Wiley-Intersience, 2nd ed., USA, 1986, p. 566.

12. G. N. Vanderplaats *Numerical Optimization Techniques for Engineering Design*, McGraw-Hill, New York, USA, 1984, p. 333.

13. S. Güths, Anémomètre a effet peltier et fluxmètre thermique: conception et réalisation. application à l'etude de la convection naturelle, *Thèse de Doctorat*, Université d'Artois, France, (1994).

14. S. M. M. Lima e Silva, T. H. Ong, and G. Guimarães, Thermal properties estimation of polymers using only one active surface, *Journal of the Brazilian Society of Mechanical Sciences and Engineering* - RBCM, pp. 09-14, **XXV**, Jan, (2003).

15. S. M. M. Lima e Silva, *Experimental Technique Development for Determining Thermal Properties Diffusivity and Thermal Conductivity of Non-metallic Materials Using Only One Active Surface*, Doctorate Thesis (in portuguese), Federal University of Uberlândia, Uberlândia, Brazil, 2000, p. 115.

16. NPL, Certificate of calibration: thermal conductivity of a pair of polythene specimens, *Technical Report* No. X2321/90/021, England, (unpublished), (1991).